Automation of One-Loop Calculations with Golem/Samurai

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XXXV International Conference on Theoretical Physics
“Matter to the Deepest”
Ustroń, Poland, September 12-18, 2011
1 Motivation & Introduction
   - Automation
   - Integrand-level OPP Reduction
   - Algebraic Approach

2 One-loop calculations with GoSam

3 Examples and Applications
Recent Progress on $2 \to 4(5)$

\begin{align*}
    pp &\to W + 3 \text{ jets} \quad \text{Rocket (2009), Blackhat (2009)} \\
    pp &\to t\bar{t}b\bar{b} \quad \text{Denner-Dittmaier (2009), Helac-NLO (2009)} \\
    pp &\to Z(\gamma) + 3 \text{ jets} \quad \text{Blackhat (2010)} \\
    pp &\to t\bar{t}jj \quad \text{Helac-NLO (2010)} \\
    pp &\to W^+ W^- b\bar{b} \quad \text{Denner-Dittmaier (2010), Helac-NLO (2010)} \\
    pp &\to W^+ W^+ jj \quad \text{Rocket (2010)} \\
    pp &\to W(Z) + 4 \text{ jets} \quad \text{Blackhat (2011)} \\
    pp &\to b\bar{b}b\bar{b} \quad \text{Golem/Samurai (2011)} \\
    pp &\to W^+ W^- jj \quad \text{Rocket (2011)} 
\end{align*}

Several methods/codes “available on the market”
**Introduction**

**Automation is unavoidable** as the complexity of the calculations increases

- number of diagrams
- number of terms in each diagram
- combination of different initial states
- etc. . .

**Virtues of Automation** (for Calculations of Scattering Amplitudes):

- **Optimization / Self-organization**
- **Avoid human mistakes**
- **Process-independent** techniques

Automation has been successfully implemented at the tree-level!
OPP Reduction - Important Points

1. The functional form of the OPP master formula is universal (process independent).

2. The only information required in order to extract the coefficients of the master integrals is the knowledge of the numerical value of the numerator function for a finite set of values \( \{q_i\} \) of the integration momentum.

3. The method does not provide any recipe for the calculation of the numerator functions.

4. The reduction becomes particularly simple if we choose \( \{q_i\} \) such that sets of denominators \( D_i \) vanish ("cuts").

G.O., Papadopoulos, Pittau
At the One-Loop level, amazing work has been done towards Automation:

- "Proofs of concept": HELAC-NLO, MadLoop
  Helac-nlo: Bevilacqua, Czakon, van Hameren, Papadopoulos, Pittau, Worek
  MadLoop: Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau
  - Upgrade of tree-level tools combined with integrand level reduction
  - Fully integrated with real radiation and subtraction terms to produce finite results

- There is also an “algebraic way” to Automation
  FeynArts/FormCalc/LoopTools: T. Hahn
  - Generate unintegrated amplitudes with Feynman diagrams
  - Manipulate and simplify them
  - Perform the reduction
Main features of the “Algebraic Way”:

- Amplitudes generated with Feynman diagrams
- Algebraic manipulations are allowed before starting the numerical integration
- The generation of numerators is executed separately from the numerical reduction
- Optimization: grouping of diagrams, smart caching
- Control over sub-parts of the computation (move in/out subsets of diagrams)
- Algebra in dimension $d$, different schemes

Great flexibility in the reduction
Choice between different algorithms at runtime
Automation with GoSam

Golem/Samurai

Algebraic generation of d-dimensional integrands via Feynman diagrams

Reduction at the Integrand Level: d-dimensional extension of OPP reduction

Target: provide an automated tool for stable evaluation of one-loop matrix elements

- be general/model independent (QCD, EW, BSM)
- interface with existing tools like MadEvent, Sherpa, PowHEG, ...
- build upon open source tools only (i.e. Samurai, Golem95, QGraf, Form, Spinney, Haggies, QCDLoop, OneLOop)
- support open standards

Giovanni Ossola (City Tech)
Cullen, Greiner, Heinrich, Luisoni, Mastrolia, G.O., Reiter, Tramontano

An automated amplitude generation based on Feynman diagrams (distributed as a python package)

- FORM
  J.A.M. Vermaseren, (1991)
- QGRAF
  P. Nogueira, (1993)
- Haggies
  T. Reiter, (2009)
- Spinney
  Cullen, Koch-Janusz, Reiter, (2010)
Diagram Reduction

Default Option: **Samurai**
Mastrolia, G.O., Reiter, Tramontano (2010)

- OPP Reduction Algorithm G.O., Papadopoulos, Pittau (2007)
- d-dimensional extension Ellis, Giele, Kunszt, Melnikov (2008)
- Coefficients of Polynomials via DFT Mastrolia et al. (2008)
- Model-independent Computation of the full Rational Term

Other options available (at runtime):

**Golem95**
Binoth, Cullen, Guillet, Heinrich, Kleinschmidt, Pilon, Reiter, Rodgers (2008)

**Tensorial Integrand-level Reduction**
Heinrich, G.O., Reiter, Tramontano (2010)
Last Step: multiply all coefficients with the corresponding Master Integral

QCDloop
(Ellis, Zanderighi)

OneLOop
(A. van Hameren)

Golem95C
(Cullen, Guillet, Heinrich, Kleinschmidt, Pilon, Reiter, Rodgers)
Upgrade of Golem95 library, real and complex masses supported
http://projects.hepforge.org/~golem/95/

New!
LoopTools
(T. Hahn)

New!
PJFry
(V. Yundin)
Virtual Corrections

Standard GoSam reduction:

- Generation of $A(q)$
- Reduction to One-Loop MI
- Computation of MI

This process is fully automated

Golem-2.0
SAMURAI
OneLOop
QCDLoop
Golem95C
Preparation of the “card”: we use as example $u \bar{d} \rightarrow \bar{s} c e^- \bar{\nu}_e \mu^+ \nu_\mu$

\begin{verbatim}
in= u,d~
out= nmu, mu+, e-, ne~, s~, c
model=smdiag
    models can be added via FeynRules (Duhr) or LanHEP (Semenov)
order=gw,4,4; order=gs,2,4
zero=mB,mC,mS,mU,mD,me,mmu
one=gs,e
helicities=+++++++ 
extensions=samurai, dred
\end{verbatim}
Building the code: check the details before the run
Building the code: check the details before the run

Diagram 36
\[ S' = S_{Q \rightarrow q^+ (-k_3+k_2+k_1-k_4)}^{(2)} d(k_2), \text{rk} = 2 \]

Diagram 3
\[ S' = S_{Q \rightarrow q^- (-k_3+k_2+k_1-k_4)} d(k_2), \text{rk} = 3 \]

Diagram 31
\[ S' = S_{Q \rightarrow -q^- (-k_3+k_1-k_4)}^{(3)} d(k_2), \text{rk} = 2 \]
A walk through GoSam

Building the code: Spinney+ Haggies
A walk through GoSam

**Execution:** all the code is ready

```
giovanni@giovanni-VPCEB27FX:~/HepForge/golem/golem-2.0/examples/loop4

File Edit View Search Terminal Help

- Codegen
- Diagrams-1.log
- Makefile.source
- process.hh
- topotree.py
- common
- Network
- doc
- matrix
- pyxotree.log
- topotree.pyc
- config.sh
- func.txt
- model
- pyxotree.tex
- topovirt.log
- diagrams-0.hh
- helicity0
- model.hh
- pyxotree.tex
- topovirt.py
- diagrams-0.log
- Makefile
- model.py
- pyxotree.tex
- topovirt.pyc
- diagrams-1.hh
- Makefile.conf
- model.pyc
- topotree.log

```

```
giovanni@giovanni-VPCEB27FX:~/HepForge/golem/golem-2.0/examples/loop4/matrix$

File Edit View Search Terminal Help

- debug
- Makefile
- matrix.f90
- test.f90
- ltest.dat
- matrix.a
- test.exe

```

NLO/LO, finite part $-15.91575134226371$

NLO/LO, single pole $7.587050691447690$

NLO/LO, double pole $-5.333333333333456$

```
timing (ms) = 5.3999999999999995
```

Table 8 of arXiv:1104.2327: Melia, Melnikov, Rontsch, Zanderighi
Alternative Path: the “Tensorial Way”

Tensorial Reconstruction at the Integrand Level

Heinrich, G.O., Reiter, Tramontano JHEP 1010:105,2010

In this work:

- We **tested** the methods for the **detection of instabilities**
- We proposed a “**rescue-system**” alternative to higher precision routines
- We proposed an **optimized reconstruction method**

Idea: **tensorial reconstruction** performed at the **integrand level** by means of a **sampling in the integration momentum**.

\[ \mathcal{N}(q) = \sum_{r=0}^{R} C_{\mu_1 \ldots \mu_r} q_{\mu_1} \ldots q_{\mu_r} \implies \hat{\mathcal{N}}(q) \]

\( \hat{\mathcal{N}}(q) \) is the “**reconstructed numerator**” written as a tensor – numerically identical to the initial \( \mathcal{N}(q) \) –
Use the decomposition of the numerator function $N(\bar{q})$ after determining all coefficients

$$N(\bar{q}) = \sum_{i<<m}^{n-1} \Delta_{ijk\ell m}(\bar{q}) \prod_{h\neq i,j,k,\ell,m}^{n-1} \bar{D}_h + \sum_{i<<\ell}^{n-1} \Delta_{ijk\ell}(\bar{q}) \prod_{h\neq i,j,k,\ell}^{n-1} \bar{D}_h +$$

$$+ \sum_{i<<k}^{n-1} \Delta_{ijk}(\bar{q}) \prod_{h\neq i,j,k}^{n-1} \bar{D}_h + \sum_{i<j}^{n-1} \Delta_{ij}(\bar{q}) \prod_{h\neq i,j}^{n-1} \bar{D}_h + \sum_{i}^{n-1} \Delta_{i}(\bar{q}) \prod_{h\neq i}^{n-1} \bar{D}_h$$

1. **Global** ($N = N$)-test
2. **Local** ($N = N$)-test
3. **Power-test**

Are those methods **reliable** in detecting **unstable phase space points**?
We approach a kinematic configuration which can lead to large cancellations.

Fermion loop with two massless and two massive vector particles:

\[ \gamma(p_1) \gamma(p_2) \gamma^*(p_3) \gamma^*(p_4) \]

\[
p_{1,2} = (E, 0, 0, \pm E) \quad p_{1,2}^2 = 0
\]

\[
p_{3,4} = (E, 0, \pm Q \sin \theta, \pm Q \cos \theta)
\]

\[
p_{3,4}^2 = m^2
\]

\[
E = \sqrt{m^2 + Q^2}
\]

The Gram-det vanishes when \( Q \to 0 \) (\( m \) and \( \theta \) are fixed):

\[
\det G = 32 E^4 Q^2 \sin^2 \theta
\]
Approaching the Gram - II

- Tensorial (double)
- Standard (quadruple)
- Standard (double)

- Pole coefficient
  - Single pole
  - Double pole

- Discriminant
  - N=N test
  - Power test
  - Local N=N test

- Relative timing

Amplitude [arb. units]

\( m^2 \text{det } G/\text{det } S \)
Ways to use the Tensorial Reduction

- “Rescue-system”
  - Unstable points will be automatically reprocessed using the tensorial decomposition + tensor integrals with Golem 95
  - Tensorial “master” integrals appears to be less costly than multi-precision routines

- “Hybrid method” for improved timing
  - The reduction of $\hat{N}(q)$ can be faster than that of $\hat{N}(q)$

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<th># Lines</th>
<th>Time ratio “hybrid”/standard</th>
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</thead>
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<tr>
<td>N</td>
<td></td>
</tr>
<tr>
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<td>1.3</td>
</tr>
<tr>
<td>10</td>
<td>1.1</td>
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<tr>
<td>1000</td>
<td>0.30</td>
</tr>
<tr>
<td>10000</td>
<td>0.27</td>
</tr>
</tbody>
</table>
**Example: Alternative reduction paths**

Samurai/Tensorial Reduction/Golem95

\[ u\bar{u} \rightarrow d\bar{d} \]

1. Evaluation with Samurai, sampling of diagram groups
2. Evaluation with Samurai, sampling of individual diagrams
3. Tensorial Reconstruction + Reduction of numetens with Samurai
4. Evaluation with Golem95

<table>
<thead>
<tr>
<th>Method</th>
<th>finite part</th>
<th>single pole</th>
<th>double pole</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-3.433053565229151</td>
<td>-14.62937842683104</td>
<td>-5.333333333333338</td>
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<tr>
<td>2</td>
<td>-3.433053565229129</td>
<td>-14.62937842683102</td>
<td>-5.3333333333333342</td>
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<tr>
<td>3</td>
<td>-3.433053565229163</td>
<td>-14.62937842683104</td>
<td>-5.3333333333333342</td>
</tr>
<tr>
<td>4</td>
<td>-3.433053565229146</td>
<td>-14.62937842683102</td>
<td>-5.3333333333333332</td>
</tr>
</tbody>
</table>
Rational Terms

\[ A = C_4 + C_3 + C_2 + C_1 + \mathcal{R} \]

Rational Terms: \( \mathcal{R} = R_1 + R_2 \)

\( R_1 \) originates from the reconstruction of \( d \)-dimensional denominators \( \bar{D}_i \).

\( R_2 \) originates from the \( d \)-dimensional part of the numerator function \( \bar{N}(\bar{q}) \)

\[ \bar{N}(\bar{q}) = N(q) + \tilde{N}(\bar{q}^2, q, \epsilon) \]

In the original OPP approach, \( R_2 \) is computed using an ad-hoc set of tree-level like Feynman Rules

G.O., Papadopoulos, Pittau (2007)
Draggiotis, Garzelli, Malamos, Pittau (2009-2011)
GoSam and Rational Terms $R_2$

GoSam offers different options for calculation of $R_2$

Thanks to the fact that we generate analytic expressions for the $d$-dimensional numerator function $\bar{N}(\bar{q})$

- **implicit**: $R_2$ terms are kept in the numerator and reduced at runtime using the $d$-dimensional decomposition of the numerator
- **explicit**: $R_2$ terms are calculated analytically (without entering in the numerical decomposition)
- **only**: only the $R_2$ term is kept in the final result (this option does not require any additional libraries)
- **off**: all $R_2$ terms are set to zero
Calculations tested with Golem/Samurai

- $u \bar{d} \rightarrow W^+ s\bar{s} \rightarrow e^+ \nu_e s\bar{s}$
- $u \bar{d} \rightarrow W^+ gg \rightarrow e^+ \nu_e gg$
- $d \bar{d} \rightarrow Z gg \rightarrow e^+ e^- gg$
- $u \bar{d} \rightarrow W^+ b\bar{b} \rightarrow e^+ \nu_e b\bar{b}$ also with massive $b$'s
- $u \bar{d} \rightarrow W^+ g \rightarrow e^+ \nu_e g$ EW corrections
- $e^+ e^- \rightarrow Z \rightarrow d \bar{d} g$
- $\gamma \gamma \rightarrow \gamma \gamma \gamma \gamma$
- $q \bar{q} \rightarrow b\bar{b} b\bar{b}$
- $g g \rightarrow b\bar{b} b\bar{b}$
- $u \bar{d} \rightarrow W^+ W^+ s\bar{c} \rightarrow e^+ \nu_e \mu^+ \nu_\mu s\bar{c}$
- $u \bar{u} \rightarrow W^+ W^- \bar{c}c \rightarrow e^- \bar{\nu}_e \mu^+ \nu_\mu \bar{c}c$
- $u \bar{d} \rightarrow W^+ W^- s\bar{c} \rightarrow e^- \bar{\nu}_e \mu^+ \nu_\mu s\bar{c}$
- plus a large number of $2 \rightarrow 2$ processes

- $d + g \rightarrow d + g$
- $d + \bar{d} \rightarrow t + \bar{t}$
- $b + g \rightarrow H + b$
- $u + \bar{u} \rightarrow g + \gamma$
- $u + g \rightarrow u + \gamma$
- $g + g \rightarrow g + \gamma$
- $g + g \rightarrow g + g$
- $g + g \rightarrow Z + g$
- $g + g \rightarrow Z + Z$
- $g + g \rightarrow W^+ + W^-$
- (...and more)
**Example:** \( gg \to gg \)

<table>
<thead>
<tr>
<th></th>
<th>Golem/Samurai</th>
<th>hep-ph/0609054</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
<td>14.120983050796795</td>
<td>14.120983050796804</td>
</tr>
<tr>
<td>NLO/LO finite</td>
<td>-124.02475579423496</td>
<td>-124.02475579423495</td>
</tr>
<tr>
<td>NLO/LO ( 1/\epsilon )</td>
<td>44.003597347101028</td>
<td>44.003597347101035</td>
</tr>
<tr>
<td>NLO/LO ( 1/\epsilon^2 )</td>
<td>-12.0000000000000002</td>
<td>-12.0000000000000000</td>
</tr>
</tbody>
</table>

Comparison with: hep-ph/0609054 Binoth, Guillet, Heinrich
Example: $pp \rightarrow W^+ W^+ jj$

$$u\bar{d} \rightarrow \bar{c}se^+ \nu_e \mu^+ \nu_\mu$$

Golem/Samurai (NLO/LO):
- finite part 23.3596455167118
- single pole 13.6255429251954
- double pole -5.33333333333343

Comparison with Melia, Melnikov, Rontsch, Zanderighi
**Example: GoSam + Sherpa**

*W + jet at LHC (14 TeV)*

(W+1 jet: $p_\perp$ of 1st jet)

(Thanks to Jennifer Archibald and Gionata Luisoni)
 Production of a heavy neutral MSSM Higgs boson and a $\bar{b}b$ pair in gluon fusion.
The loop contains two squarks and two neutralinos

**Real Masses**

**Complex Masses**

Golem95C – arXiv:1101.5595
Example: GoSam and MSSM Neutralino

\[ pp \rightarrow \chi_1^0 \chi_1^0 \]

(Thanks to Gavin Cullen and Nicolas Greiner)
**Conclusions: Golem/Samurai**

There are many valuable approaches/codes to One-Loop Calculations

**GoSam is a flexible and broadly applicable tool**

- it is based on Feynman diagrams
- it uses a d-dimensional reduction (no additional techniques required for rational terms)
- it will be publicly available, as soon as we complete the testing
- it uses some of the best techniques on the market

We look forward to interacting/interfacing with other tools

More results soon!
EXTRA SLIDES
4-dim identity at the integrand level for $N(q)$ in terms of 4-dim $D_i$

$$N(q) = \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} \left[ d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i$$

$$+ \sum_{i_0 < i_1 < i_2}^{m-1} \left[ c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i$$

$$+ \sum_{i_0 < i_1}^{m-1} \left[ b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] \prod_{i \neq i_0, i_1}^{m-1} D_i + \sum_{i_0}^{m-1} \left[ a(i_0) + \tilde{a}(q; i_0) \right] \prod_{i \neq i_0}^{m-1} D_i$$

$\tilde{d}(q)$, $\tilde{c}(q)$, $\tilde{b}(q)$, $\tilde{a}(q)$ are “spurious” terms that vanish upon integration.
4-dim identity at the integrand level for $N(q)$ in terms of 4-dim $D_i$

\[
N(q) = \sum_{i_0 < i_1 < i_2 < i_3}^{m-1} \left[ d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3) \right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i \\
+ \sum_{i_0 < i_1 < i_2}^{m-1} \left[ c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2) \right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i \\
+ \sum_{i_0 < i_1}^{m-1} \left[ b(i_0 i_1) + \tilde{b}(q; i_0 i_1) \right] \prod_{i \neq i_0, i_1}^{m-1} D_i + \sum_{i_0}^{m-1} \left[ a(i_0) + \tilde{a}(q; i_0) \right] \prod_{i \neq i_0}^{m-1} D_i
\]

Rational Terms

In this approach, Rational Terms require a separate computation

G.O., Papadopoulos, Pittau (2007)
Draggiotis, Garzelli, Malamos, Pittau (2009-2011)
Identity in d-dimensions: \( q \rightarrow \bar{q} \)

Ellis, Giele, Kunszt, Melnikov (2008), Melnikov, Schulze (2010)
Mastrolia, G.O., Reiter, Tramontano (2010)

Reconstruct directly d-dimensional denominators

\[
N(\bar{q}) = \sum_{i<<m}^{n-1} \Delta_{ijk\ell m}(\bar{q}) \prod_{h\neq i,j,k,\ell,m} \bar{D}_h + \sum_{i<<\ell}^{n-1} \Delta_{ijk\ell}(\bar{q}) \prod_{h\neq i,j,k,\ell} \bar{D}_h + \\
+ \sum_{i<<k}^{n-1} \Delta_{ijk}(\bar{q}) \prod_{h\neq i,j,k} \bar{D}_h + \sum_{i<j}^{n-1} \Delta_{ij}(\bar{q}) \prod_{h\neq i,j} \bar{D}_h + \sum_{i}^{n-1} \Delta_{i}(\bar{q}) \prod_{h\neq i} \bar{D}_h
\]

1. Denominators in d-dimensions: \( D_h \rightarrow \bar{D}_h \)
2. \( N(\bar{q}) \) is d-dimensions: \( N(q) \rightarrow N(q, \mu^2) \)
3. The polynomials in the coefficients have a more complicated structure
Identity in d-dimensions: Coefficients

- Add a spurious pentagon term in $\mu^2$

$$\Delta_{ijk\ell m}(\bar{q}) = c_{5,0}^{(ijklm)} \mu^2$$

- The coefficients have a more complicated structure

**Box in 4-dimensions**

$$\Delta_{ijk\ell}(q) = c_{4,0} + c_{4,1} \tilde{F}(q)$$

**Box in d-dimensions**

$$\Delta_{ijk\ell}(\bar{q}) = c_{4,0} + c_{4,2} \mu^2 + c_{4,4} \mu^4 + (c_{4,1} + c_{4,3} \mu^2) \tilde{F}(q)$$