Nonorthogonal neutrino mixing matrix and neutrino oscillations

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Standard Approach

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{-i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{-i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{-i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{-i\delta} & c_{23}c_{13} \end{pmatrix}$$

$$P_{\alpha\to\beta,\alpha\neq\beta} = \Omega_{\alpha\beta} \pm \mathbf{Y} \sum_\gamma \epsilon_{\alpha\beta\gamma}, \quad \{\alpha\beta\gamma\} = \{e\mu\tau\}$$

$$P_{\alpha\to\alpha} = 1 + \Omega_{\alpha\alpha}$$

$$\mathbf{A_{CP}} = \frac{P(\nu_\mu \to \nu_e) - P(\bar\nu_\mu \to \bar\nu_e)}{P(\nu_\mu \to \nu_e) + P(\bar\nu_\mu \to \bar\nu_e)}$$

$$\mathbf{A_{T}} = \frac{P(\nu_\mu \to \nu_e) - P(\nu_e \to \nu_\mu)}{P(\nu_\mu \to \nu_e) + P(\nu_\mu \to \nu_e)}$$

$$\Omega_{\alpha\beta} = -4\left[R^{21}_{\alpha\beta}\sin^2\Delta_{21} + R^{31}_{\alpha\beta}\sin^2\Delta_{31} + R^{32}_{\alpha\beta}\sin^2\Delta_{32}\right]$$

$$\mathbf{Y} = -2\mathbf{J}\left[\sin 2\Delta_{21} - \sin 2\Delta_{31} + \sin 2\Delta_{32}\right] = -8\mathbf{J}\sin\boldsymbol{\Delta_{21}}\sin\boldsymbol{\Delta_{31}}\sin\boldsymbol{\Delta_{32}}$$

$$\mathbf{J} \equiv I^{12}_{e\mu} = \frac{1}{8}\sin\delta\cos^{\mathbf{2}}\boldsymbol{\Theta_{13}}\sin\mathbf{2}\boldsymbol{\Theta_{12}}\sin\mathbf{2}\boldsymbol{\Theta_{23}}\sin\boldsymbol{\Theta_{13}}$$

$$\boldsymbol{\Delta_{ab}} = \delta\mathbf{m^2_{ab}}[eV]\frac{\mathbf{L[km]}}{\mathbf{E[GeV]}}, \quad \delta\mathbf{m^2_{ab}} = \mathbf{m^2_a} - \mathbf{m^2_b}$$

$$W^{ab}_{\alpha\beta} = U_{\alpha a}U_{\beta b}U^*_{\alpha b}U^*_{\beta a}$$

$$R^{ab}_{\alpha\beta} = Re\left[W^{ab}_{\alpha\beta}\right]$$

$$I^{ab}_{\alpha\beta} = Im\left[W^{ab}_{\alpha\beta}\right]$$

- J large: all three mixing angles large:
  Θ_{12} - O.K. (three such solutions for $\nu_\odot$)
  Θ_{23} - O.K.
  Θ_{13}: no, $\Theta_{13} \leq 0.05$;
- $\sin\delta$ substantial;
- all Δ_{ij} large: $\frac{L}{E}$, $\delta m^2_{ab} \Rightarrow$ long-baseline exp., LMA MSW

then ...

“ ... I will aasume that nature is kind and ....”

... and many papers come

Choice (consistent with CHOOZ, LMA MSW, SK):

$$\Delta m^2_{21} = 5 \cdot 10^{-5}\ eV^2$$

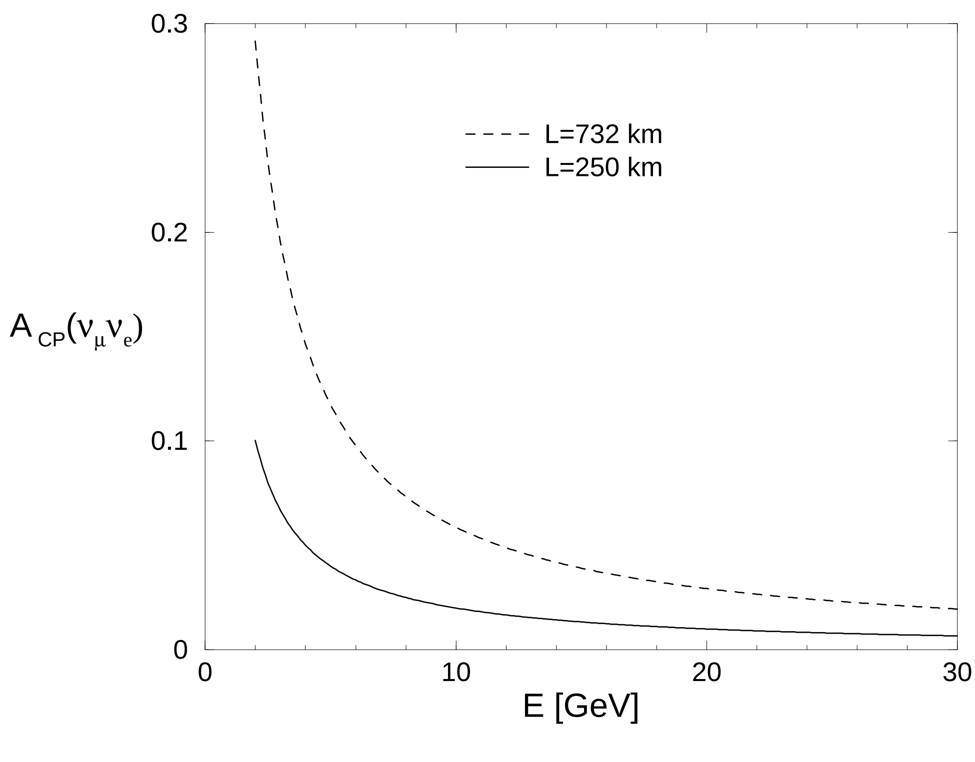
$$\Delta m^2_{31} = 3 \cdot 10^{-3}\ eV^2$$

$$\Theta_{12} \simeq 35^o$$

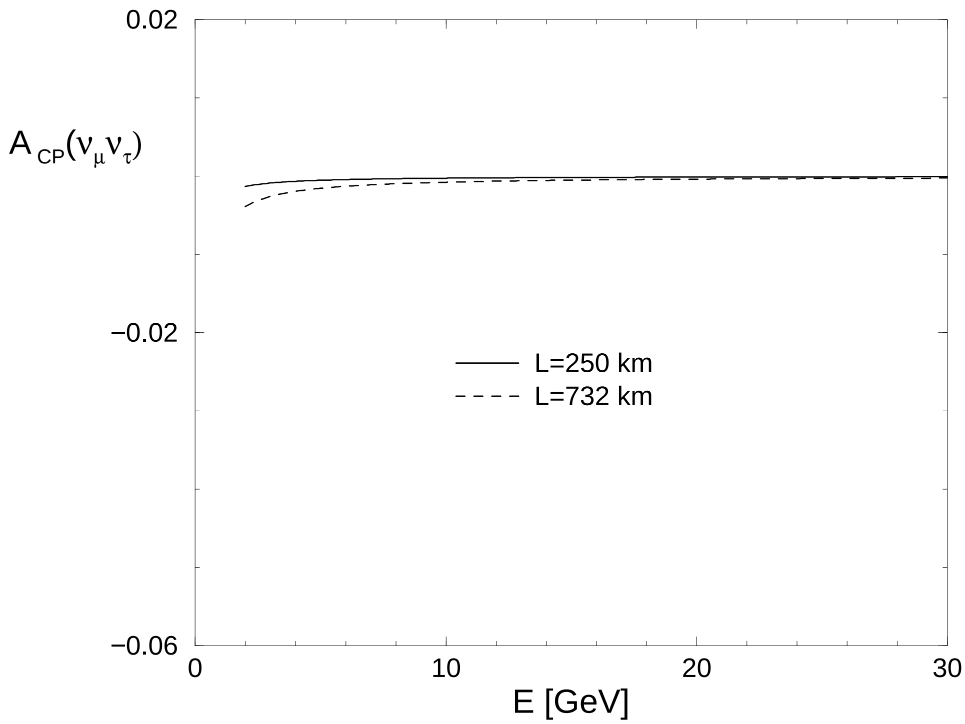
$$\Theta_{23} \simeq 40^o$$

$$\Theta_{13} \simeq 5^o$$

$$\delta \simeq \pm 90^o$$



[the same choice of parameters]



Nonorthogonal Light Neutrinos

$$\nu_\alpha = \sum_{i=1}^{3} U_{\alpha i} \nu_i$$

We know: $m_{\nu_i} \leq \mathcal{O}(eV)$, left-handed neutrinos

$$\begin{aligned} |\nu_e\rangle &= U_{e1}|\nu_1\rangle + U_{e2}|\nu_2\rangle + U_{e3}|\nu_3\rangle \\ |\nu_\mu\rangle &= \ldots \end{aligned}$$

U is 3×3 unitary, weak neutrino states are orthogonal

however, ...

$$\begin{pmatrix} \nu_{\mathbf{L}} \\ \nu_L^c \end{pmatrix} = \begin{pmatrix} \mathbf{U} & \mathbf{V} \\ V' & U' \end{pmatrix} \begin{pmatrix} \nu \\ N \end{pmatrix}_L$$

Such a construction is a consequence of introducing general Dirac-Majorana mass Lagrangian to the theory:

$$L_{D-M} = (\bar{\nu}_L \bar{\nu}_L^c) \begin{pmatrix} 0 & m_D \\ m_D^T & m_R \end{pmatrix} \begin{pmatrix} \nu_R^c \\ \nu_R \end{pmatrix}$$

$m_D << m_R \Rightarrow \mathbf{U} << \mathbf{V}$

and minor modification:

$$\begin{aligned} |\nu_e\rangle &= \mathbf{U_{e1}}|\nu_1\rangle + \mathbf{U_{e2}}|\nu_2\rangle + \mathbf{U_{e3}}|\nu_3\rangle + \mathbf{V_{e1}}|N_1\rangle \ldots \\ |\nu_\mu\rangle &= \ldots \end{aligned}$$

$U_\nu U_\nu^\dagger = 1 \Rightarrow UU^\dagger + VV^\dagger = 1$

$$\nu_\alpha = \sum_{i=1}^{3} U_{\alpha i} \nu_i$$

remains the basic initial equation (but with not unitary U) how much U can be nonunitary? (how large V can be?) Let us consider minimal extension with U_ν of 4×4 dimension

$$U_\nu = \mathbf{R_{34}R_{24}R_{14}R_{23}R_{13}R_{12}},$$

where rotations take place in 4 dimensional space spanned by four massive neutrino states, e.g.

$$R_{12} = \begin{pmatrix} c_{12} & s_{12}e^{i\gamma} & 0 & 0 \\ -s_{12}e^{-i\gamma} & c_{12} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

e.g. V. Barger et.al, PRD59,113010

For $m_4 >> m_i$ $R_{34}R_{24}R_{14}$ rotations are small. Let us take $s_{14} \equiv \boldsymbol{\epsilon_1 << 1}$, $s_{24} \equiv \boldsymbol{\epsilon_2 << 1}$ and $s_{34} \equiv \boldsymbol{\epsilon_3 << 1}$. Then we can expand U_ν to get

$$U_\nu = \begin{pmatrix} \left(U(\epsilon_{\mathbf{i}}) \right) & \begin{matrix} \epsilon_{\mathbf{1}} \\ \epsilon_{\mathbf{2}} \\ \epsilon_{\mathbf{3}} \end{matrix} \\ g(\epsilon_{\mathbf{i}}) & 1 - \frac{1}{2}(|\epsilon_{\mathbf{1}}|^2 + |\epsilon_{\mathbf{2}}|^2 + |\epsilon_{\mathbf{3}}|^2) \end{pmatrix}$$

$\epsilon_i \to 0 \Rightarrow U(\epsilon_i) \to U_{3\times 3}$ and $g(\epsilon_i) \to 0$.

$$\nu_\alpha = \sum_{i=1}^{3} U(\epsilon_{\mathbf{i}})_{\alpha i} \nu_i$$

Constraints on ϵ_i:

$$\begin{aligned} \left(VV^\dagger\right)_{ee} &= \sum_{i=heavy} |V_{ei}|^2 \equiv |\epsilon_{\mathbf{1}}|^{\mathbf{2}} \leq \mathbf{0.0054}, \\ |\left(VV^\dagger\right)_{e\mu}| &= |\epsilon_{\mathbf{1}}\epsilon_{\mathbf{2}}| \leq \mathbf{10^{-4}}, \\ |\left(VV^\dagger\right)_{\mu\tau}| &= |\epsilon_{\mathbf{2}}\epsilon_{\mathbf{3}}| \leq \mathbf{10^{-2}}. \end{aligned}$$

$$P_{\nu_\alpha \to \nu_\beta} = N_\alpha^2 N_\beta^2 \left\{ \left(\delta_{\alpha\beta} - \left| \left(VV^\dagger\right)_{\alpha\beta} \right| \right)^2 \right.$$

$$- 4 \sum_{a>b} \tilde{R}^{ab}_{\alpha\beta} \sin^2 \Delta_{ab}$$

$$- 8 \tilde{I}^{12}_{\alpha\beta} \sin \Delta_{21} \sin \Delta_{31} \sin \Delta_{32}$$

$$\left. - 2 \left[A^{(1)}_{\alpha\beta} \sin 2\Delta_{31} + A^{(2)}_{\alpha\beta} \sin 2\Delta_{32} \right] \right\},$$

$$\tilde{R}(\tilde{I})^{ab}_{\alpha\beta} = Re(Im) \left[W^{ab}_{\alpha\beta} \right],$$

$$W^{ab}_{\alpha\beta}(\epsilon_{\mathbf{i}}) = U_{\alpha a} U_{\beta b} U^*_{\alpha b} U^*_{\beta a},$$

$$N_\alpha^2 = \frac{1}{1 - (VV^\dagger)_{\alpha\alpha}},$$

$$A^{(i)}_{\alpha\beta}(\epsilon_{\mathbf{i}}) = Im \left[U^*_{\alpha i} U_{\beta i} \left(VV^\dagger\right)_{\alpha\beta} \right],$$

unitary U ($J \equiv I_{e\mu}^{12}$):

$$\begin{aligned} I_{\alpha\beta}^{ab} &= -I_{\alpha\beta}^{ba} = -I_{\beta\alpha}^{ab} = I_{\beta\alpha}^{ba} \\ I_{\alpha\beta}^{ab} &= Im\left[U_{\alpha a}U_{\beta b}U_{\alpha b}^{*}U_{\beta a}^{*}\right] \end{aligned}$$

U is not unitary:

$$\begin{aligned} I_{\alpha\beta}^{12} &= -I_{\alpha\beta}^{32} - Im\left(U_{\alpha 2}^{*}U_{\beta 2}\left(VV^{\dagger}\right)_{\alpha\beta}\right) \\ I_{\alpha\beta}^{21} &= -I_{\alpha\beta}^{31} - Im\left(U_{\alpha 1}^{*}U_{\beta 1}\left(VV^{\dagger}\right)_{\alpha\beta}\right) \\ I_{\alpha\beta}^{23} &= -I_{\alpha\beta}^{13} - Im\left(U_{\alpha 3}^{*}U_{\beta 3}\left(VV^{\dagger}\right)_{\alpha\beta}\right) \end{aligned}$$

$$A_{\alpha\beta}^{(i)}(\epsilon_{\mathbf{i}}) = Im\left[U_{\alpha i}^{*}U_{\beta i}\left(VV^{\dagger}\right)_{\alpha\beta}\right]$$

consequences: CP violating effects for

1. $\delta m_{12}^2 = 0$;
2. N=2;
3. N=3, $\delta = 0$

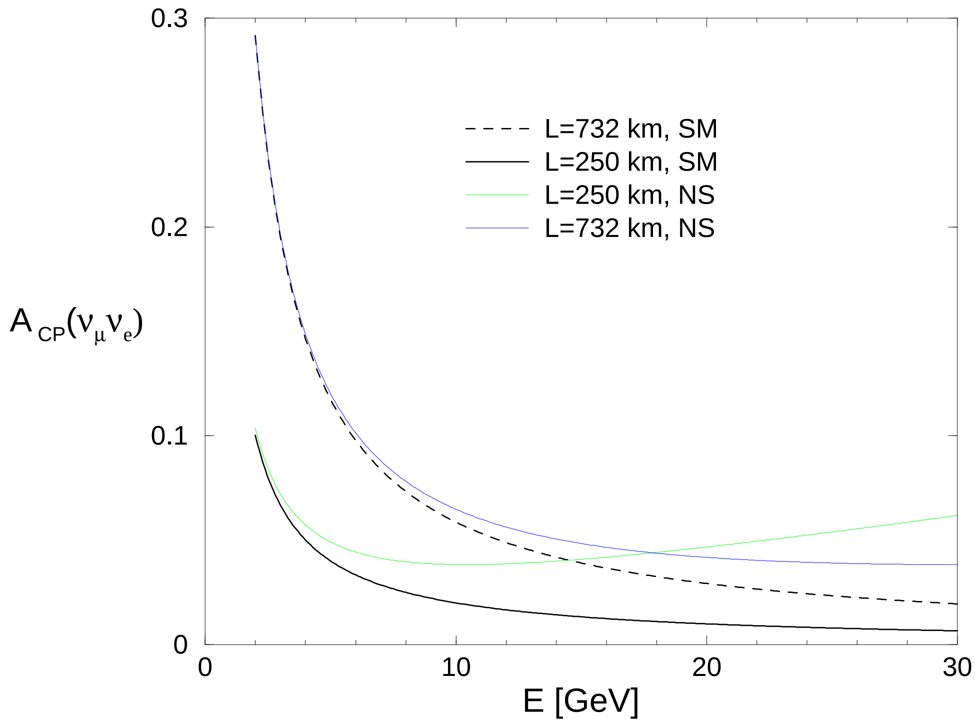
$$P_{\nu_\alpha \to \nu_\beta} \sim \left[A_{\alpha\beta}^{(1)} \sin 2\Delta_{31} + A_{\alpha\beta}^{(2)} \sin 2\Delta_{32}\right]$$

[the choice of SM neutrino parameters as before]

$\epsilon_1 = 0.01$

$\epsilon_2 = 0.01$

$\epsilon_3 = 0.1$



[the choice of SM neutrino parameters as before]

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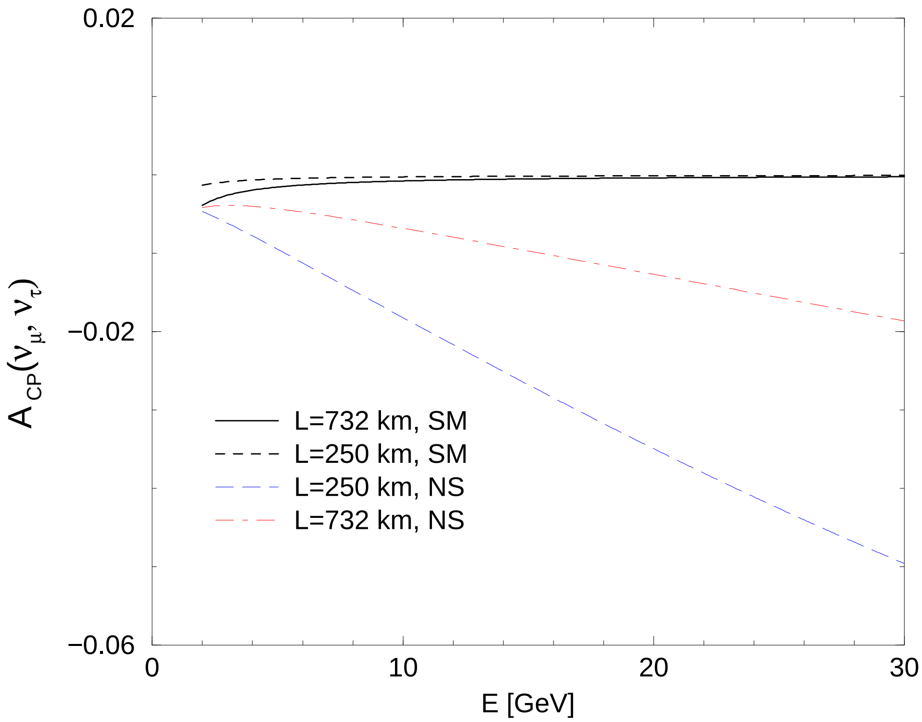
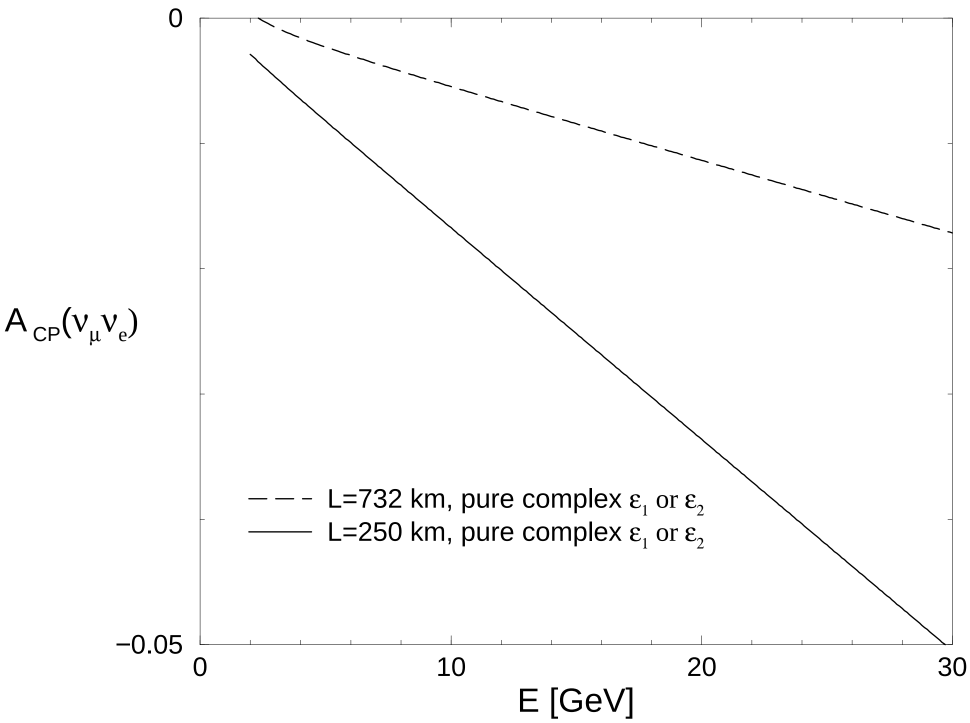


Genuine CP effects from NS phases: $\nu_\mu - \nu_e$

[the choice of SM neutrino parameters as before, but $\delta = 0$]

$\epsilon_{1(2)} = 0.01 \cdot \{1(\mathbf{i})$ or $\mathbf{i}(1)\}$

$\epsilon_3 = 0.01$

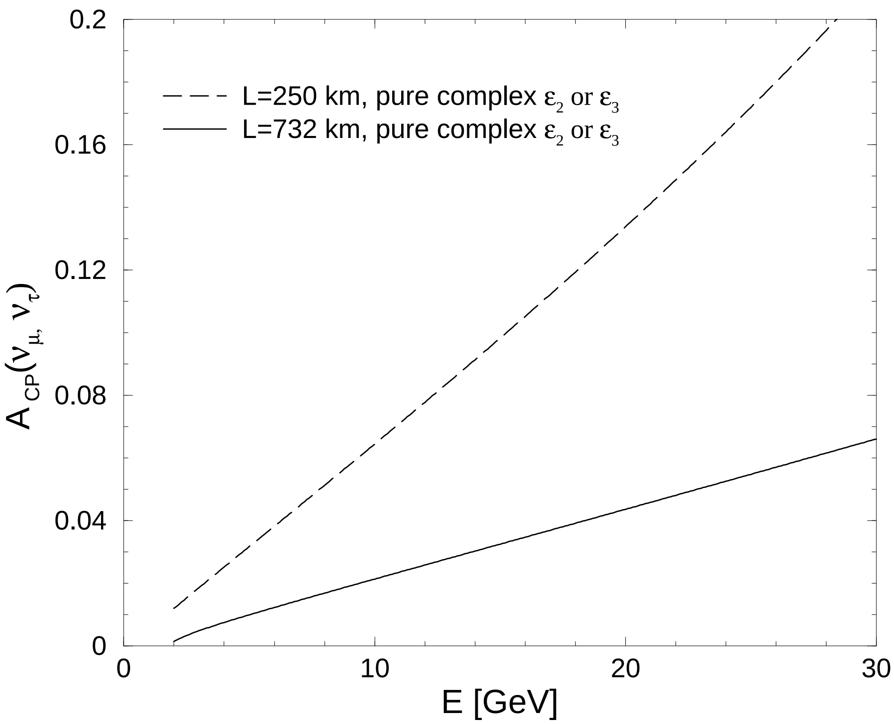


Genuine CP effects from NS phases: $\nu_\mu - \nu_\tau$

[the choice of SM neutrino parameters as before, but $\delta = 0$]

$\epsilon_3 = 10^{-4}$

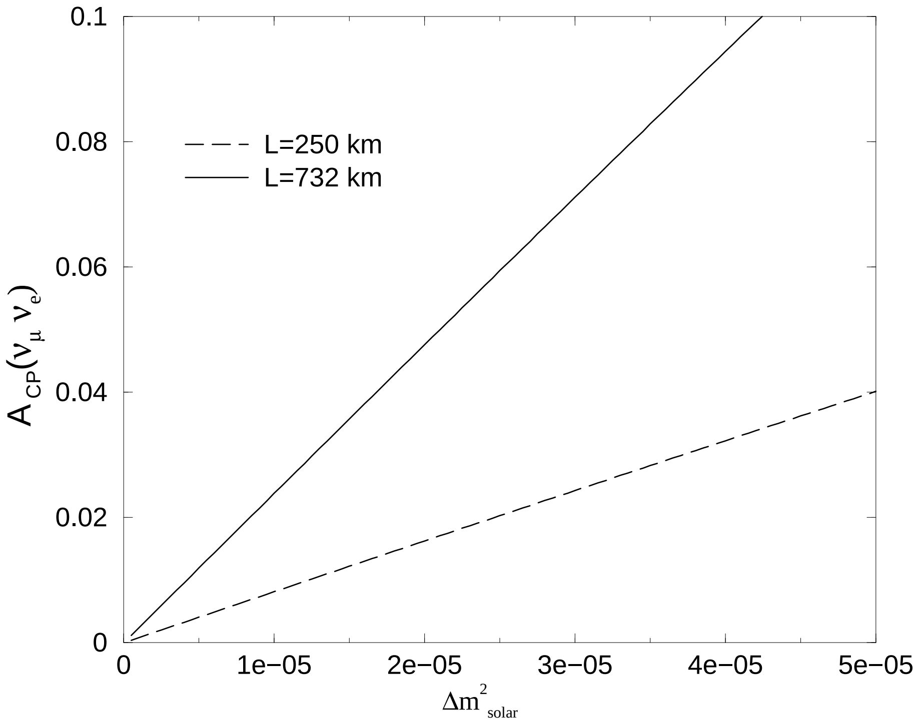
$\epsilon_{2(3)} = 0.1 \cdot \{1(\mathbf{i})$ or $\mathbf{i}(1)\}$



A_{CP} **versus** Δm^2_{12}: $\nu_\mu - \nu_e$

[the choice of SM neutrino parameters as before, E=5 GeV]

$\epsilon_i = 0$



What does **"nonorthogonality"** mean?

$$\sum_{\beta} P_{\alpha\to\beta} = 1$$

e.g.:

$$P_{ee} + P_{e\mu} + P_{e\tau} = 1$$

U not unitary:

$$U = \begin{pmatrix} \cos\Theta_1 & \sin\Theta_1 \\ -\sin\Theta_2 & \cos\Theta_2 \end{pmatrix}; \quad \Theta_2 = \Theta_1 + \epsilon$$

we get

$$\begin{aligned}
\sum_{\alpha=e,\mu} P_{e\alpha} &= P_{ee} + P_{e\mu} \\
&= 1 + 4\epsilon \sin^2\Delta_{21} \sin\Theta_1 \cos\Theta_1 \cos 2\Theta_1 \\
&+ \mathcal{O}(\epsilon^2), \\
\sum_{\alpha=e,\mu} P_{\mu\alpha} &= P_{\mu e} + P_{\mu\mu} \\
&= 1 - 4\epsilon \sin^2\Delta_{21} \sin\Theta_1 \cos\Theta_1 \cos 2\Theta_1 \\
&+ \mathcal{O}(\epsilon^2).
\end{aligned}$$

Conclusions: unusal effects

- CP-violation in short-baseline experiments?
- SM phase $\delta = 0$ and CP effects?
- Δm^2_{sol} not in the LMA MSW region and CP effects?
- cancelations between SM and NS contributions?
- it can hapened that CP-violation effects would be detected with a strength larger than predicted in an unitary neutrino mixing approach: heavy neutrino mixing could be an answer
- if searching for CP violating effects agree with data then (in some cases) better bounds on the $\left(VV^\dagger\right)_{\alpha\beta}$ factors can be found