

Muon decay to one-loop order

in the LR-symmetric model

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M. Czakon, J. Gluza, J. Hejczyk, hep-ph/0205303, to appear in Nucl. Phys. B.

1. Introduction to the LRM

The model has been constructed in 1973-1974,

Pati, Salam, Senjanovic, Mohapatra

$SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ gauge group

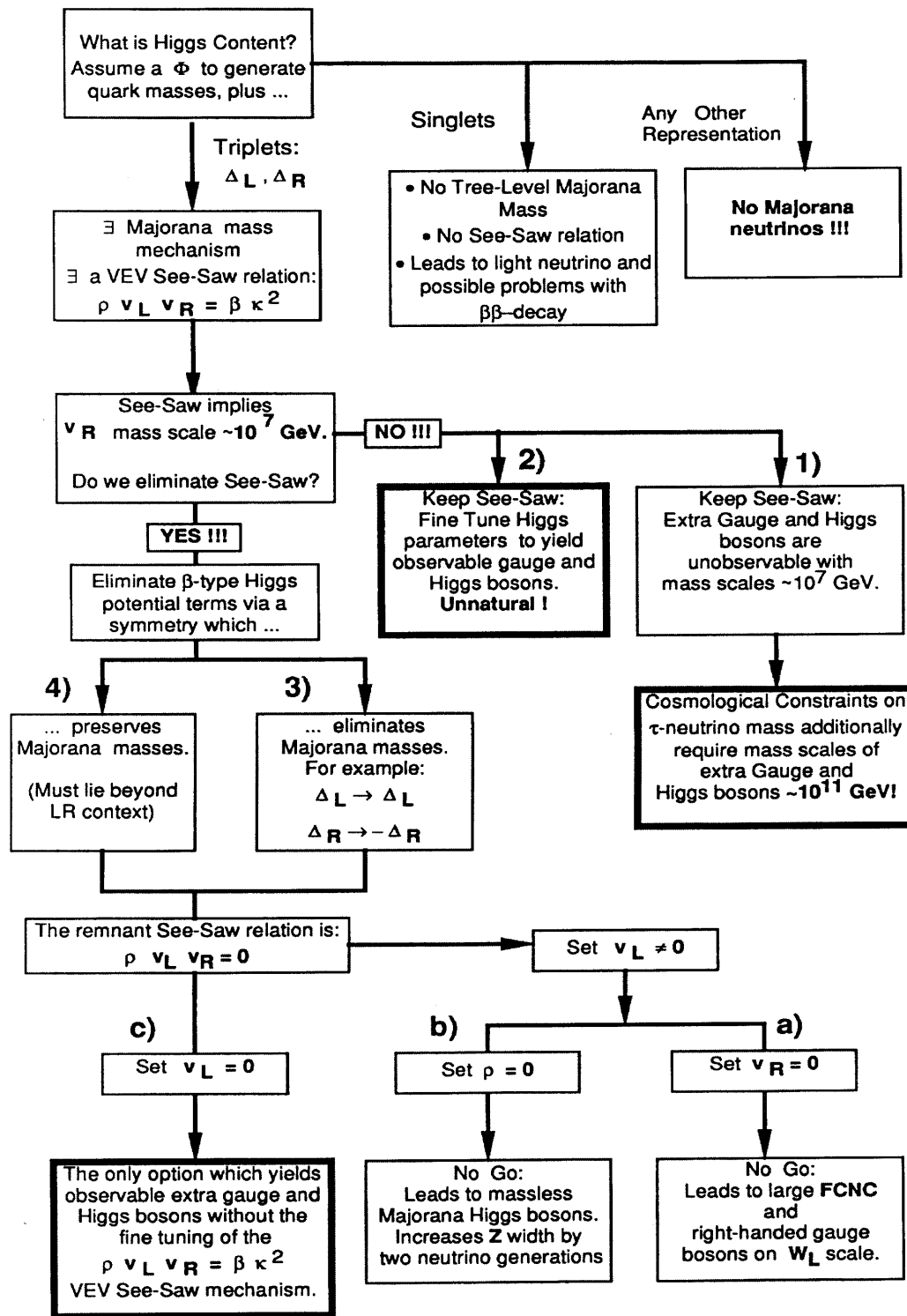
(i) restore lepton-quark symmetry to the weak interactions:

$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}, \quad \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad \begin{pmatrix} u_R \\ d_R \end{pmatrix}$$

(ii) Interpretation of the hypercharge as the difference of the baryon and lepton numbers:

$$Q = T_{3L} + T_{3R} + \frac{B - L}{2}$$

$$\begin{array}{ccc} W_L^\pm, W_L^0 & & W_1^\pm, W_2^\pm \\ W_R^\pm, W_R^0 & \rightarrow [SSB?] & Z_1, Z_2 \\ B^0 & & \gamma \end{array}$$



$$\begin{aligned}
 \mathcal{L}_{Higgs} = & \\
 & -\mu_1^2 \text{Tr}[\Phi^\dagger \Phi] - \mu_2^2 (\text{Tr}[\tilde{\Phi} \Phi^\dagger] + \text{Tr}[\tilde{\Phi}^\dagger \Phi]) \\
 & -\mu_3^2 (\text{Tr}[\Delta_L \Delta_L^\dagger] + \text{Tr}[\Delta_R \Delta_R^\dagger]) \\
 & +\lambda_1 \text{Tr}[\Phi \Phi^\dagger]^2 + \lambda_2 (\text{Tr}[\tilde{\Phi} \Phi^\dagger]^2 + \text{Tr}[\tilde{\Phi}^\dagger \Phi]^2) \\
 & +\lambda_3 (\text{Tr}[\tilde{\Phi} \Phi^\dagger] \text{Tr}[\tilde{\Phi}^\dagger \Phi]) \\
 & +\lambda_4 (\text{Tr}[\Phi \Phi^\dagger] (\text{Tr}[\tilde{\Phi} \Phi^\dagger] + \text{Tr}[\tilde{\Phi}^\dagger \Phi])) \\
 & +\rho_1 (\text{Tr}[\Delta_L \Delta_L^\dagger]^2 + \text{Tr}[\Delta_R \Delta_R^\dagger]^2) \\
 & +\rho_2 (\text{Tr}[\Delta_L \Delta_L] \text{Tr}[\Delta_L^\dagger \Delta_L^\dagger] + \text{Tr}[\Delta_R \Delta_R] \text{Tr}[\Delta_R^\dagger \Delta_R^\dagger]) \\
 & +\rho_3 (\text{Tr}[\Delta_L \Delta_L^\dagger] \text{Tr}[\Delta_R \Delta_R^\dagger]) \\
 & +\rho_4 (\text{Tr}[\Delta_L \Delta_L] \text{Tr}[\Delta_R^\dagger \Delta_R^\dagger] + \text{Tr}[\Delta_R \Delta_R] \text{Tr}[\Delta_L^\dagger \Delta_L^\dagger]) \\
 & +\alpha_1 (\text{Tr}[\Phi \Phi^\dagger] (\text{Tr}[\Delta_L \Delta_L^\dagger] + \text{Tr}[\Delta_R \Delta_R^\dagger])) \\
 & +\alpha_2 (\text{Tr}[\Phi \tilde{\Phi}^\dagger] \text{Tr}[\Delta_R \Delta_R^\dagger] + \text{Tr}[\tilde{\Phi} \Phi^\dagger] \text{Tr}[\Delta_L \Delta_L^\dagger]) \\
 & +\alpha_2^* (\text{Tr}[\Phi^\dagger \tilde{\Phi}] \text{Tr}[\Delta_R \Delta_R^\dagger] + \text{Tr}[\tilde{\Phi}^\dagger \Phi] \text{Tr}[\Delta_L \Delta_L^\dagger]) \\
 & +\alpha_3 (\text{Tr}[\Phi \Phi^\dagger \Delta_L \Delta_L^\dagger] + \text{Tr}[\Phi^\dagger \Phi \Delta_R \Delta_R^\dagger]) \\
 & +\beta_1 (\text{Tr}[\Phi \Delta_R \Phi^\dagger \Delta_L^\dagger] + \text{Tr}[\Phi^\dagger \Delta_L \Phi \Delta_R^\dagger]) \\
 & +\beta_2 (\text{Tr}[\tilde{\Phi} \Delta_R \Phi^\dagger \Delta_L^\dagger] + \text{Tr}[\tilde{\Phi}^\dagger \Delta_L \Phi \Delta_R^\dagger]) \\
 & +\beta_3 (\text{Tr}[\Phi \Delta_R \tilde{\Phi}^\dagger \Delta_L^\dagger] + \text{Tr}[\Phi^\dagger \Delta_L \tilde{\Phi} \Delta_R^\dagger]),
 \end{aligned}$$

invariant under the symmetry $\Delta_L \leftrightarrow \Delta_R$, $\Phi \leftrightarrow \Phi^\dagger$, $\beta_i = 0$.

The minimal Higgs sector consists of two triplets and one bidoublet

$$\Delta_{L,R} = \begin{pmatrix} \delta_{L,R}^+/\sqrt{2} & \delta_{L,R}^{++} \\ \delta_{L,R}^0 & -\delta_{L,R}^+/\sqrt{2} \end{pmatrix},$$

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}.$$

with vacuum expectation values allowed for the neutral particles

$$\frac{v_L}{\sqrt{2}} = \langle \delta_L^0 \rangle,$$

new HE scale : $\frac{v_R}{\sqrt{2}} = \langle \delta_R^0 \rangle,$

SM VEV scale : $\sqrt{\kappa_1^2 + \kappa_2^2}$

$$\frac{\kappa_1}{\sqrt{2}} = \langle \phi_1^0 \rangle,$$

$$\frac{\kappa_2}{\sqrt{2}} = \langle \phi_2^0 \rangle.$$

- The result is 20 real scalar fields, of which 14 are physical (the rest are Goldstone bosons):
 - 4 neutral scalars: $H_0^0, H_1^0, H_2^0, H_4^0$,
(the first can be considered to be the light Higgs of the SM at tree level),
 - 2 neutral pseudo-scalars: A_1^0, A_2^0 ,
 - 2 charged scalars: H_1^\pm, H_2^\pm ,
 - 2 doubly-charged scalars: $H_1^{\pm\pm}, H_2^{\pm\pm}$.
- see-saw mechanism for the generation of light neutrino masses, with specific SB sectors. The neutrino mass matrix

$$M_\nu = \begin{pmatrix} M_L(\nu_L) & M_D(\kappa_{1,2}) \\ M_D^T & M_R(\nu_R) \end{pmatrix}$$

with $M_L \ll M_D \ll M_R$.

$$\begin{aligned} m_N &\sim M_R \\ m_{\text{light}} &\sim M_D^2/M_R \end{aligned}$$

$M_D \sim \mathcal{O}(1) \text{ GeV} \rightarrow M_R \sim 10^{15} \text{ GeV}$, if light neutrino masses of the order of 0.1 eV.

W_R (Right-Handed W Boson) MASS LIMITS

Assuming a light right-handed neutrino, except for BEALL 82, LANGACKER 89B, and COLANGELO 91. $g_R = g_L$ assumed. [Limits in the section MASS LIMITS for W' below are also valid for W_R if $m_{\nu_R} \ll m_{W_R}$.] Some limits assume manifest left-right symmetry, *i.e.*, the equality of left- and right Cabibbo-Kobayashi-Maskawa matrices. For a comprehensive review, see LANGACKER 89B. Limits on the W_L - W_R mixing angle ζ are found in the next section. Values in brackets are from cosmological and astrophysical considerations and assume a light right-handed neutrino.

VALUE (GeV)	CL%	DOCUMENT ID	TECN	COMMENT
> 715	90	11 CZAKON	99 RVUE	Electroweak
> 310	90	12 THOMAS	01 CNTR	β^+ decay
> 137	95	13 ACKERSTAFF	99D OPAL	τ decay
>1400	68	14 BARENBOIM	98 RVUE	Electroweak, Z - Z' mixing
> 549	68	15 BARENBOIM	97 RVUE	μ decay
> 220	95	16 STAHL	97 RVUE	τ decay
> 220	90	17 ALLET	96 CNTR	β^+ decay
> 281	90	18 KUZNETSOV	95 CNTR	Polarized neutron decay
> 282	90	19 KUZNETSOV	94B CNTR	Polarized neutron decay
> 439	90	20 BHATTACH...	93 RVUE	Z - Z' mixing
> 250	90	21 SEVERIJNS	93 CNTR	β^+ decay
		22 IMAZATO	92 CNTR	K^+ decay
> 475	90	23 POLAK	92B RVUE	μ decay
> 240	90	24 AQUINO	91 RVUE	Neutron decay
> 496	90	24 AQUINO	91 RVUE	Neutron and muon decay
> 700		25 COLANGELO	91 THEO	$m_{K_L^0} - m_{K_S^0}$
> 477	90	26 POLAK	91 RVUE	μ decay
[none 540-23000]		27 BARBIERI	89B ASTR	SN 1987A; light ν_R
> 300	90	28 LANGACKER	89B RVUE	General
> 160	90	29 BALKE	88 CNTR	$\mu \rightarrow e \nu \bar{\nu}$
> 406	90	30 JODIDIO	86 ELEC	Any ζ
> 482	90	30 JODIDIO	86 ELEC	$\zeta = 0$
> 800		MOHAPATRA	86 RVUE	$SU(2)_L \times SU(2)_R \times U(1)$
> 400	95	31 STOKER	85 ELEC	Any ζ
> 475	95	31 STOKER	85 ELEC	$\zeta < 0.041$
		32 BERGSMA	83 CHRM	$\nu_\mu e \rightarrow \mu \nu_e$
> 380	90	33 CARR	83 ELEC	μ^+ decay
>1600		34 BEALL	82 THEO	$m_{K_L^0} - m_{K_S^0}$
[> 4000]		STEIGMAN	79 COSM	Nucleosynthesis; light ν_R

3. Assumptions

Tree level diagrams to the muon decay:

- exchanged W_1 ,
- exchanged W_2 ,
- exchanged charged Higgs particles.

We assume:

- no mixing in the charged gauge sector: $\kappa_2 = 0$, $\xi = 0$,
- mixings among light and heavy neutrino states are neglected

Effects of heavy neutrinos remain:

- $Z_1 N_i N_i$, $Z_2 N_i N_i$ (build up gauge-boson self energies);
- $\bar{e} N_4 H_2^-$: $m_{N_4} \frac{g}{2M_{W_2}} \frac{\kappa_1}{v_R} P_L$,
- $\bar{e} N_1 H_1^-$: $\sqrt{2} \frac{m_{N_4}}{v_R} P_R$

Nice feature: constraints on the heavy (right) sector come directly from 1-loop level!

$$\begin{aligned}
-\mathcal{L} &= \bar{c}_{LL} \bar{e} \gamma_\alpha P_L \nu_e \bar{\nu}_\mu \gamma^\alpha P_L \mu + \bar{c}_{RR} \bar{e} \gamma_\alpha P_R \nu_e \bar{\nu}_\mu \gamma^\alpha P_R \mu \\
&+ \bar{c}_{LR} \bar{e} \gamma_\alpha P_L \nu_e \bar{\nu}_\mu \gamma^\alpha P_R \mu + \bar{c}_{RL} \bar{e} \gamma_\alpha P_R \nu_e \bar{\nu}_\mu \gamma^\alpha P_L \mu.
\end{aligned}$$

where:

$$\begin{aligned}
\bar{c}_{LL} &= \frac{g^2}{2M_{W_1}^2} (\cos^2 \xi + \beta \sin^2 \xi), \\
\bar{c}_{RR} &= \frac{g^2}{2M_{W_1}^2} (\sin^2 \xi + \beta \cos^2 \xi), \\
\bar{c}_{RL} = \bar{c}_{LR} &= \frac{g^2}{2M_{W_1}^2} (-1 + \beta) \sin \xi \cos \xi.
\end{aligned}$$

$$\beta = \frac{M_{W_1}^2}{M_{W_2}^2},$$

To have neutrino mixings properly included:

$$U = \begin{pmatrix} K_L^T \\ K_R^\dagger \end{pmatrix} = \begin{pmatrix} O(1) & O(1/M_N) \\ O(1/M_N) & O(1) \end{pmatrix}.$$

$$c_{LL} = \bar{c}_{LL} (K_L^\dagger)_{ea} (K_L)_{b\mu},$$

$$c_{RR} = \bar{c}_{RR} (K_R^\dagger)_{ea} (K_R)_{b\mu},$$

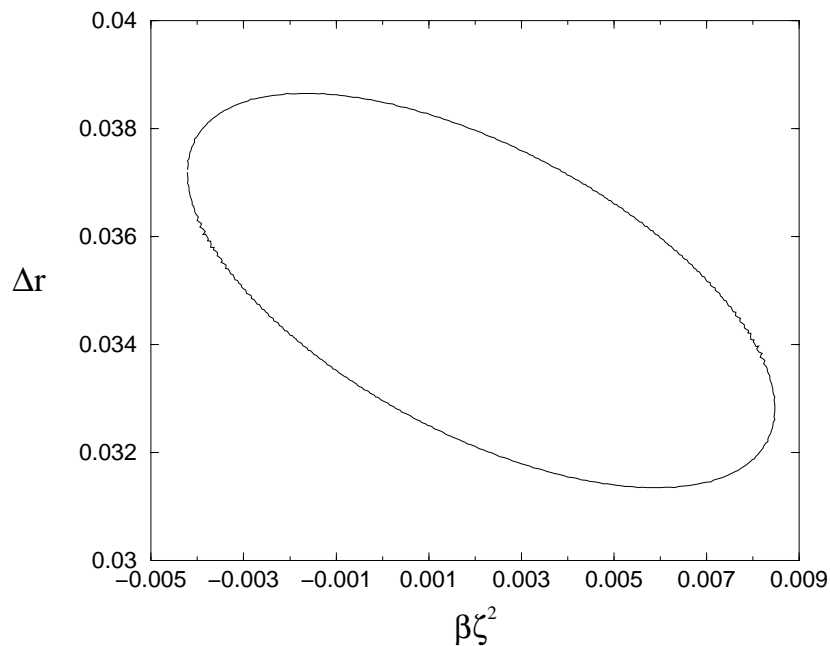
$$c_{LR} = \bar{c}_{LR} (K_L^\dagger)_{ea} (K_R)_{b\mu},$$

$$c_{RL} = \bar{c}_{RL} (K_R^\dagger)_{ea} (K_L)_{b\mu}.$$

then: $c_{LL} \gg c_{RR}, c_{LR}, c_{RL}$

and

$$\frac{G_F}{\sqrt{2}} \simeq |c_{LL}|^2 \simeq \frac{\pi\alpha}{2s_W^2 M_W^2 (1 - \Delta r)} (1 + \beta\zeta^2).$$



with the very worst bound $\zeta < .1$, it gives $\beta < 1 \rightarrow M_{W_2} \geq M_{W_1}$

without heavy neutrino mixing:

$$\begin{aligned} \frac{G_F}{\sqrt{2}} &= |c_{LL}|^2 + |c_{LR}|^2 + |c_{RL}|^2 + |c_{RR}|^2 \\ &= \frac{\pi\alpha}{2s_W^2 M_W^2 (1 - \Delta_R)} (1 + \beta^2) \end{aligned}$$

and $M_{W_2} \geq 500 \text{ GeV}$

3. Gauge Sector Renormalization

- The SSB leads to the following gauge-boson mass matrices

$$\frac{g^2}{4} \begin{pmatrix} \kappa_+^2 & -2\kappa_1\kappa_2 \\ -2\kappa_1\kappa_2 & \kappa_+^2 + 2v_R^2 \end{pmatrix},$$

$$\frac{1}{2} \begin{pmatrix} \frac{1}{2}g^2\kappa_+^2 & -\frac{1}{2}g^2\kappa_+^2 & 0 \\ -\frac{1}{2}g^2\kappa_+^2 & \frac{1}{2}g^2(\kappa_+^2 + 4v_R^2) & -2gg'v_R^2 \\ 0 & -2gg'v_R^2 & 2g'^2v_R^2 \end{pmatrix}$$

- The diagonalization introduces various mixing angles. It is possible to write the respective matrices as

$$\begin{pmatrix} W_L^\pm \\ W_R^\pm \end{pmatrix} = \begin{pmatrix} \cos \xi & \sin \xi \\ -\sin \xi & \cos \xi \end{pmatrix} \begin{pmatrix} W_{1\pm}^\pm \\ W_{2\pm}^\pm \end{pmatrix},$$

$$\begin{pmatrix} W_L^3 \\ W_R^3 \\ B \end{pmatrix} = \begin{pmatrix} c_W c & c_W s & s_W \\ -s_W s_M c - c_M s & -s_W s_M s + c_M c & c_W s_M \\ -s_W c_M c + s_M s & -s_W c_M s - s_M c & c_W c_M \end{pmatrix} \begin{pmatrix} Z_1 \\ Z_2 \\ A \end{pmatrix},$$

where:

$$c_W \equiv \cos \theta_W, \quad s_W \equiv \sin \theta_W, \quad s \equiv \sin \phi,$$

$$c_M \equiv \frac{\sqrt{\cos 2\theta_W}}{\cos \theta_W}, \quad c \equiv \cos \phi, \quad s_M \equiv \tan \theta_W$$

- $$M_{W_{1,2}}^2 = \frac{g^2}{4} \left(\kappa_+^2 + v_R^2 \mp \sqrt{v_R^4 + 4\kappa_1^2 \kappa_2^2} \right),$$

$$M_{Z_{1,2}}^2 = \frac{1}{4} \left(\left((g^2 \kappa_+^2 + 2v_R^2 (g^2 + g'^2)) \right) \right.$$

$$\left. \mp \sqrt{(g^2 \kappa_+^2 + 2v_R^2 (g^2 + g'^2))^2 - 4g^2 (g^2 + 2g'^2) \kappa_+^2 v_R^2} \right)$$

and

$$\tan 2\xi = -\frac{2\kappa_1 \kappa_2}{v_R^2}, \quad \sin 2\phi = -\frac{g^2 \kappa_+^2 \sqrt{\cos 2\theta_W}}{2 \cos^2 \theta_W (M_{Z_2}^2 - M_{Z_1}^2)}.$$

$$g = \frac{e}{\sin \theta_W}, \quad g' = \frac{e}{\sqrt{\cos 2\theta_W}}.$$

- We exploit the one-to-one correspondence of the parameters:

$$\begin{aligned} g & \quad e, \\ g' & \quad M_{W_1}, \\ \kappa_1 & \rightarrow M_{W_2}, \\ \kappa_2 & \quad M_{Z_1}, \\ v_R & \quad M_{Z_2}. \end{aligned}$$

Adopt an on-shell scheme in which we renormalize the self-energies of the gauge-bosons by requiring that they vanish at the physical mass squared. The electric charge is renormalized as in the SM.

- The renormalization parameter that is of particular interest is the Weinberg angle.

By expanding to first order, we end up with

$$\delta s_W^2 = 2c_W^2 \frac{(\delta M_{Z_2}^2 + \delta M_{Z_1}^2) - (\delta M_{W_2}^2 + \delta M_{W_1}^2)}{(M_{Z_2}^2 + M_{Z_1}^2) - (M_{W_2}^2 + M_{W_1}^2)}$$

$$+ \frac{1}{2} \frac{(M_{W_2}^2 + M_{W_1}^2)(\delta M_{Z_2}^2 + \delta M_{Z_1}^2) + (M_{Z_2}^2 + M_{Z_1}^2)(\delta M_{W_2}^2 + \delta M_{W_1}^2)}{\left((M_{Z_2}^2 + M_{Z_1}^2) - (M_{W_2}^2 + M_{W_1}^2)\right)^2}$$

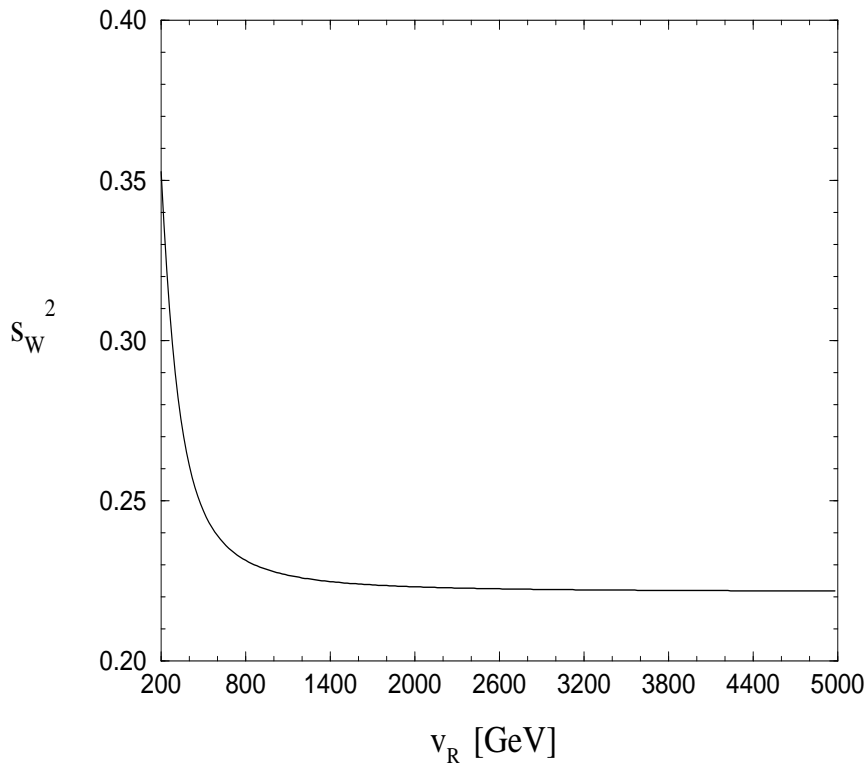
$$- \frac{1}{2} \frac{(2M_{Z_1}^2 + M_{Z_2}^2)\delta M_{Z_1}^2 + (2M_{Z_2}^2 + M_{Z_1}^2)\delta M_{Z_2}^2}{\left((M_{Z_2}^2 + M_{Z_1}^2) - (M_{W_2}^2 + M_{W_1}^2)\right)^2}.$$

The denominator is expressible through the heavy breaking scale only

$$\left((M_{Z_2}^2 + M_{Z_1}^2) - (M_{W_2}^2 + M_{W_1}^2)\right) = \frac{g^2}{2c_M^2 c_W^2} v_R^2.$$

In the SM:

$$\delta s_W^2 = c_W^2 \left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2} \right)$$



in our model s_W^2 is expressible by M_{W_1}, M_{Z_1} , and a heavy scale v_R ,

SM:

$$s_W^2 = 1 - \frac{M_{W_1}^2}{M_{Z_1}^2}$$

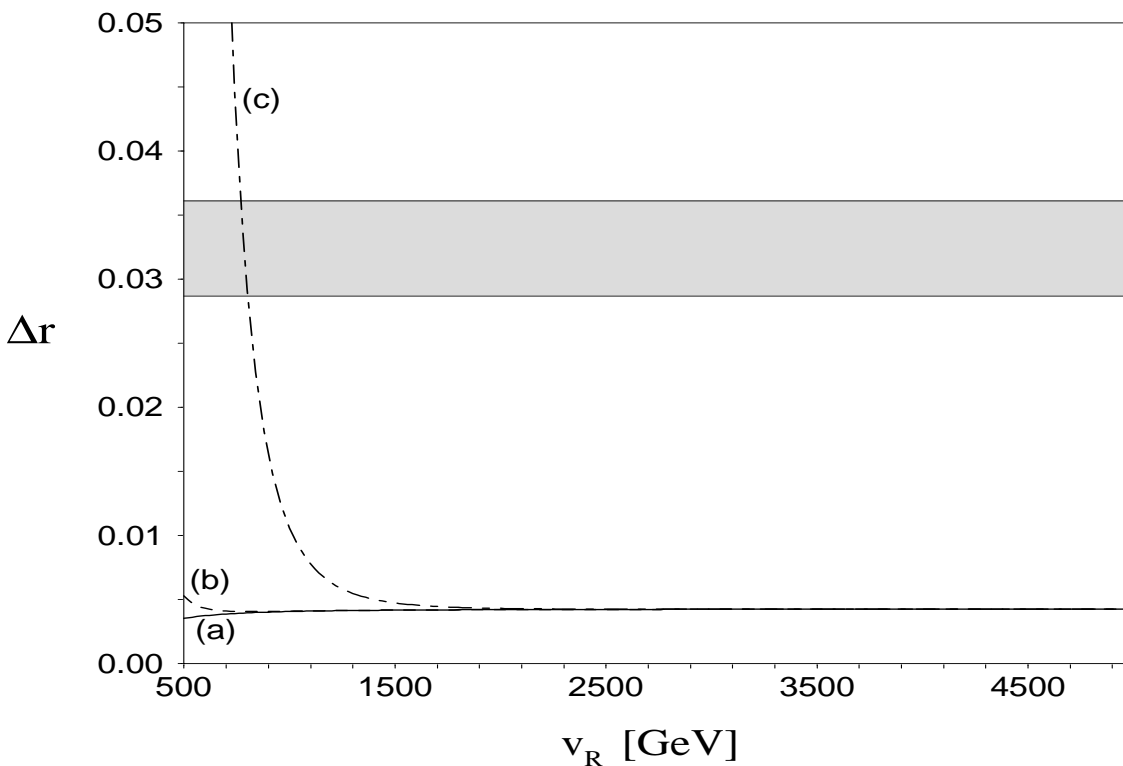
We include the change of s_W^2 from the SM to the LRM in Δr ,

$$\frac{e^2}{(8M_W^2 s_W^2)_{SM}} (1 + \Delta r) = \frac{e^2}{(8M_{W_1}^2 s_W^2)_{LR}} (1 + \Delta r_{LR})$$

then:

$$\begin{aligned} \Delta r &= \frac{(s_W^2)_{SM}}{(s_W^2)_{LRSM}} \left(\frac{-\Pi_W^T(0) - \delta M_W^2}{M_W^2} + 2 \frac{\delta e}{e} \right. \\ &\quad \left. - \frac{\delta s_W^2}{s_W^2} + \delta_V + \delta_B \right) \\ &\quad - \frac{(s_W^2)_{LRSM} - (s_W^2)_{SM}}{(s_W^2)_{LRSM}}, \end{aligned}$$

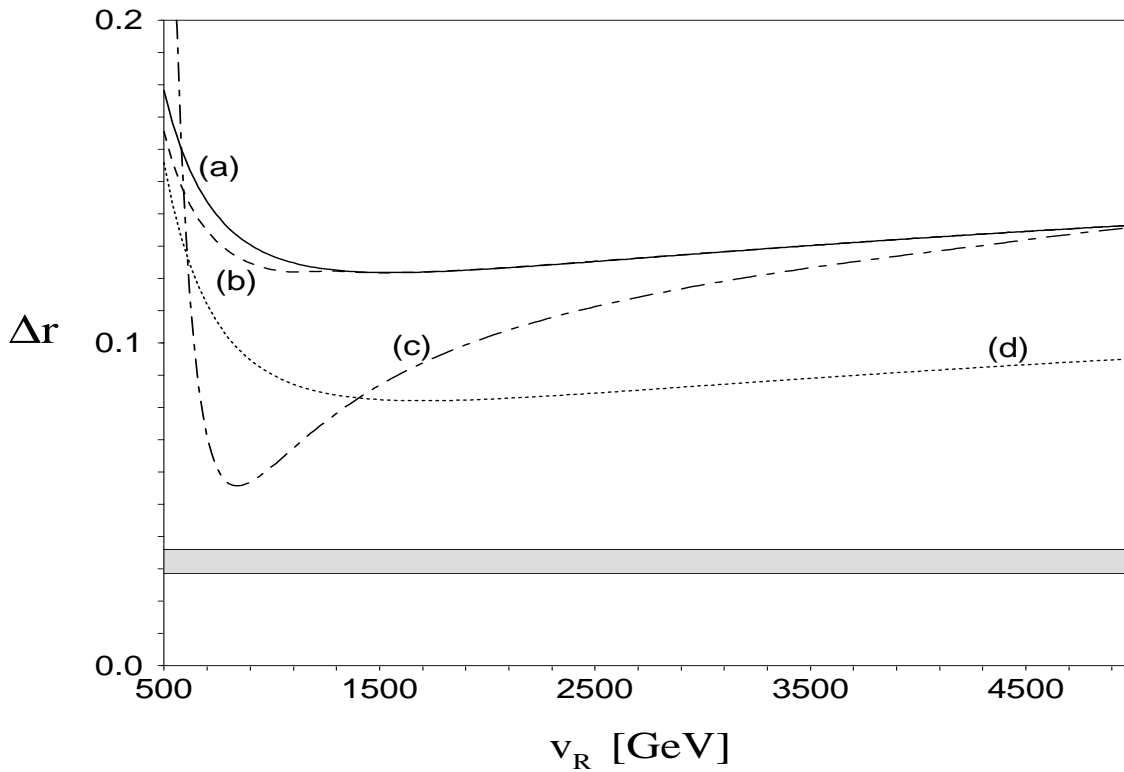
4. Results and Discussion



Contribution of box diagrams to Δr . With increasing v_R heavy particles decouple and the lines aim at the SM contribution. The (a) line is for (three heavy neutrinos) $m_N = 100$ GeV; (b) is for $m_N = 500$ GeV; (c) is for $m_N = 2$ TeV. The gray area shows the experimentally allowed values of Δr .

$$m_N = \sqrt{2}h_M v_R \leq \sqrt{2}v_R$$

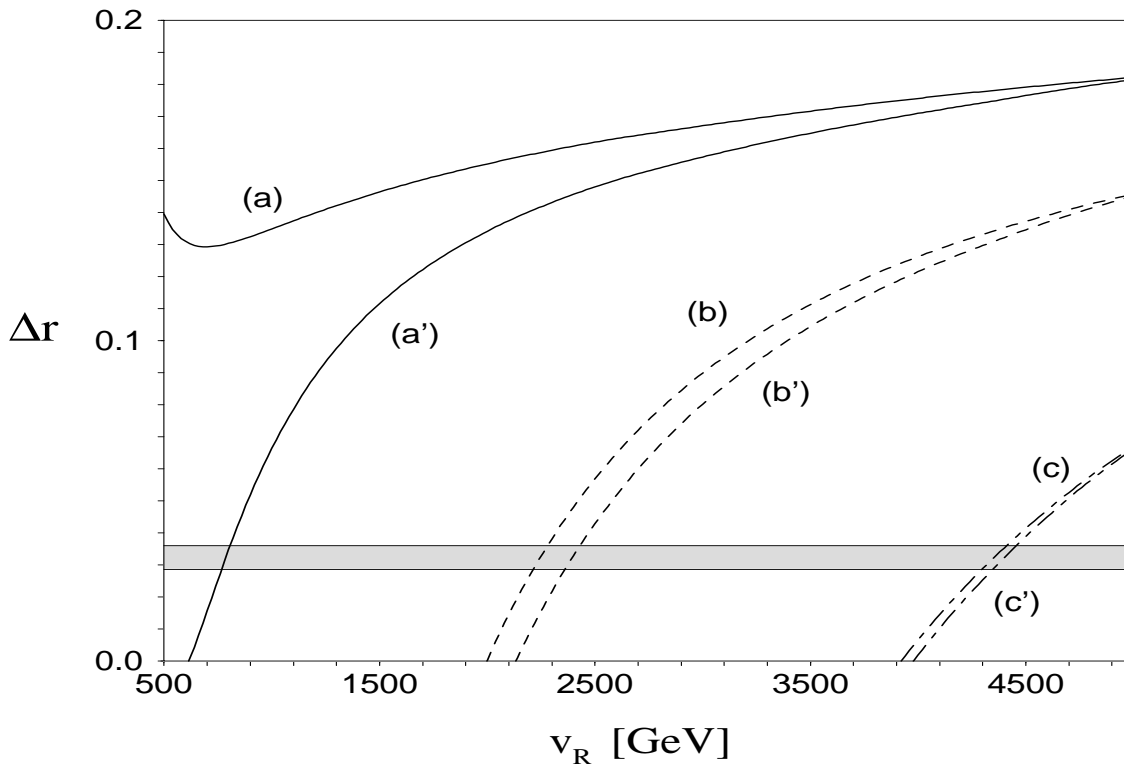
“Natural” scalar potential:



The (a) line is for (three heavy neutrinos) $m_N = 100$ GeV; (b) is for $m_N = 500$ GeV; (c) is for $m_N = 2$ TeV.

$$\begin{aligned}
 M_{H_a} &\equiv M_{H_1^0} = M_{H_3^0} = M_{A_1^0} = M_{A_2^0} \\
 &= M_{H_1^+} = M_{H_2^+} = M_{\delta_L^{++}} = v_R/\sqrt{2}, \\
 M_{H_b} &\equiv M_{H_2^0} = M_{\delta_R^{++}} = \sqrt{2}v_R, \\
 M_{H_0^0} &= \sqrt{2}\kappa_1.
 \end{aligned}$$

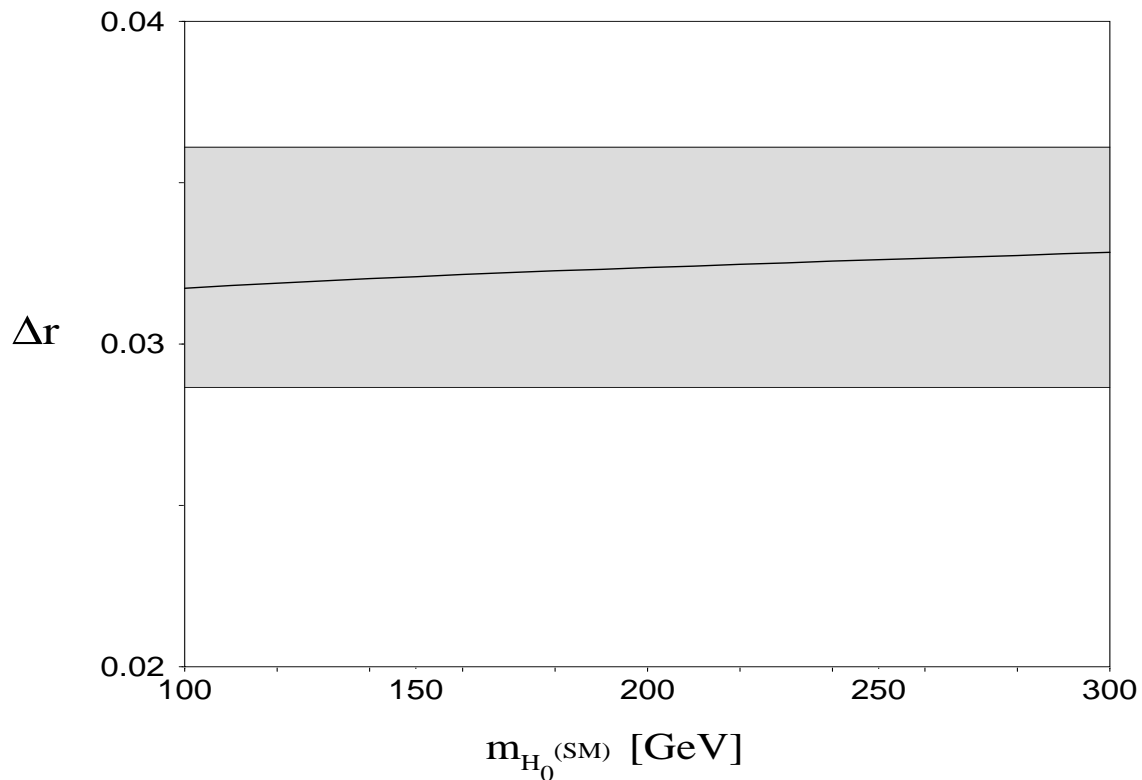
All Higgs particle masses equal (apart from H_0^0):



Sets with and without primes show results for three heavy neutrino masses with $m_N = 100$ GeV and $m_N = 2$ TeV respectively.

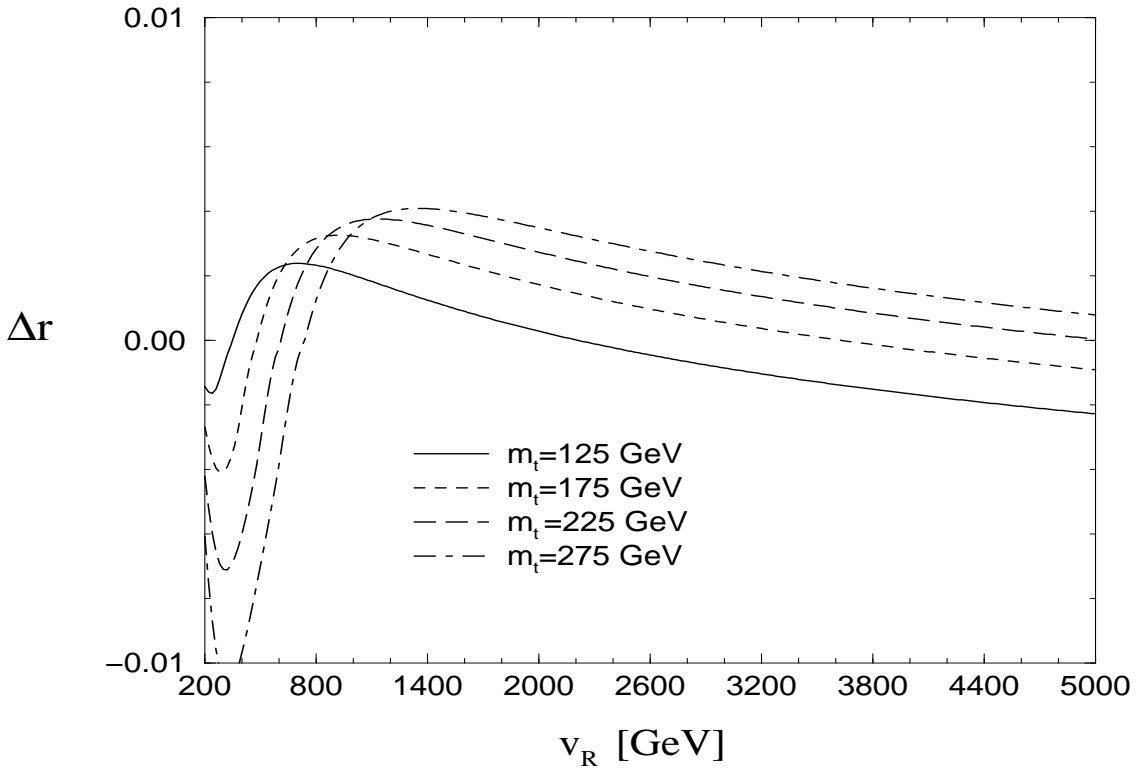
for $M_H \simeq 2v_R$ (and large v_R), Δr can be accommodated,
 remark: fine tuned masses

Variation of $m(H_0^0)$ is negligible:



Δr as function of the lightest Higgs scalar mass $M_{H_0^0}$. The gray area shows the experimentally allowed values of Δr and the heavy particle spectrum is chosen to fit approximately to this region, namely, $v_R = 2390$ GeV, $m_N = 2$ TeV, $m_H = 5$ TeV.

m_t is effectively light and is not predictable by Δr in LRM:



The contribution of the third quark family to Δr as function of v_R for different top quark masses.

Leading terms:

$$(\Delta\rho)_{SM} \simeq \frac{m_t^2}{M_W^2}$$

$$(\Delta\rho)_{LRM} \simeq \frac{m_t^2}{M_{W_2}^2 - M_{W_1}^2}$$

6. Conclusions and Outlook

LRM can be fitted to muon decay width, however:

- heavy neutrino masses and Higgs masses must be very tuned to get proper Δr :
→ low energy data ? ...
- corrections coming from SM particles within LR model do not constitute a structure equivalent to their SM structure, e.g. top quark and $\Delta\rho$, the lightest Higgs;
- consequences:
Czakon, Gluza, Jegerlehner, Zrałek, EPJ, C13, 275 (2000),
also Lynn & Nardi, NPB381, 1992, ...
- LRM gives a possibility to study many effects: RH currents, W_2 , Z_2 , δ^{++} , δ^+ , N , further questions in 1-loop level:

$$\begin{aligned}g & \quad e, \\g' & \quad M_{W_1}, \\ \kappa_1 & \rightarrow M_{W_2}, \\ \kappa_2 & \quad M_{Z_1}, \\ v_R & \quad M_{Z_2}.\end{aligned}$$

$$g_L \neq g_R, v_L \neq 0, \text{ non-finite QED with } \xi \neq 0, \dots$$